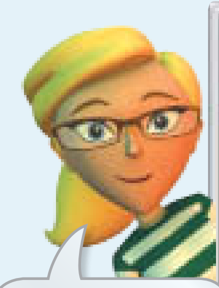
7-3 Proving Triangles Similar

Common Core State Standards
G-SRT.B.5 Use . . . similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **Also G-GPE.B.5**
MP 1, MP 3, MP 4

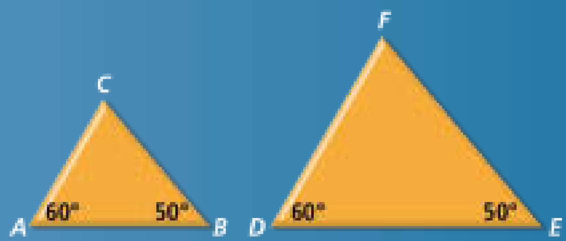
Objectives To use the AA ~ Postulate and the SAS ~ and SSS ~ Theorems
To use similarity to find indirect measurements



You've already learned how to decide whether two polygons are similar. This is a special case of that problem.

SOLVE IT! Getting Ready!

Are the triangles similar? How do you know? (Hint: Use a centimeter ruler to measure the sides of each triangle.)



MATHEMATICAL PRACTICES

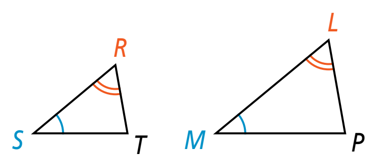
In the Solve It, you determined whether the two triangles are similar. That is, you needed information about all three pairs of angles and all three pairs of sides. In this lesson, you'll learn an easier way to determine whether two triangles are similar.

Lesson Vocabulary
- indirect measurement

Essential Understanding You can show that two triangles are similar when you know the relationships between only two or three pairs of corresponding parts.

take note

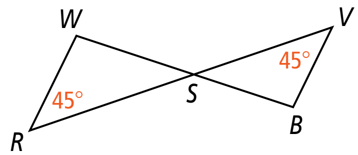
Postulate 7-1 Angle-Angle Similarity (AA ~) Postulate

Postulate	**If . . .**	**Then . . .**
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.	$\angle S \cong \angle M$ and $\angle R \cong \angle L$ 	$\triangle SRT \sim \triangle MLP$

What do you need to show that the triangles are similar?
To use the AA ~ Postulate, you need to prove that two pairs of angles are congruent.

Problem 1 Using the AA ~ Postulate

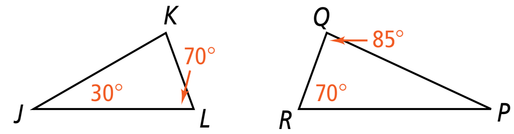
Are the two triangles similar? How do you know?



A $\triangle RSW$ **and** $\triangle VSB$

$\angle R \cong \angle V$ because both angles measure 45°.
$\angle RSW \cong \angle VSB$ because vertical angles are congruent.
So, $\triangle RSW \sim \triangle VSB$ by the AA ~ Postulate.

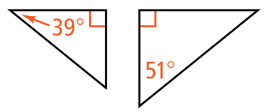
B $\triangle JKL$ **and** $\triangle PQR$



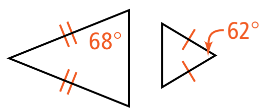
$\angle L \cong \angle R$ because both angles measure 70°.
By the Triangle Angle-Sum Theorem,
$m\angle K = 180 - 30 - 70 = 80$ and
$m\angle P = 180 - 85 - 70 = 25$. Only one pair of angles is congruent. So, $\triangle JKL$ and $\triangle PQR$ are *not* similar.

Got It? **1.** Are the two triangles similar? How do you know?

a.



b.



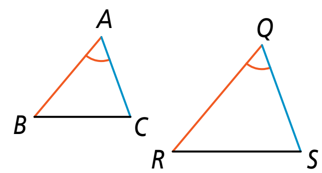
Here are two other ways to determine whether two triangles are similar.

take note

Theorem 7-1 Side-Angle-Side Similarity (SAS ~) Theorem

Theorem
If an angle of one triangle is congruent to an angle of a second triangle, and the sides that include the two angles are proportional, then the triangles are similar.

If . . .
$\frac{AB}{QR} = \frac{AC}{QS}$ and $\angle A \cong \angle Q$



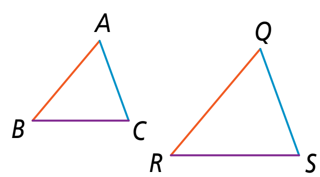
Then . . .
$\triangle ABC \sim \triangle QRS$

You will prove Theorem 7-1 in Exercise 35.

Theorem 7-2 Side-Side-Side Similarity (SSS ~) Theorem

Theorem
If the corresponding sides of two triangles are proportional, then the triangles are similar.

If . . .
$\frac{AB}{QR} = \frac{AC}{QS} = \frac{BC}{RS}$



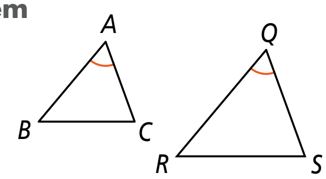
Then . . .
$\triangle ABC \sim \triangle QRS$

You will prove Theorem 7-2 in Exercise 36.

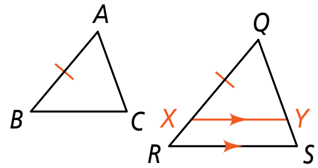
Proof Proof of Theorem 7-1: Side-Angle-Side Similarity Theorem

Given: $\frac{AB}{QR} = \frac{AC}{QS}$, $\angle A \cong \angle Q$

Prove: $\triangle ABC \sim \triangle QRS$



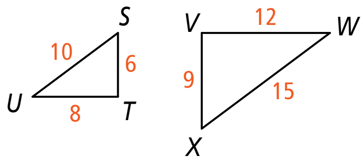
Plan for Proof: Choose X on $\overline{RQ}$ so that $QX = AB$. Draw $\overleftrightarrow{XY} \parallel \overline{RS}$. Show that $\triangle QXY \sim \triangle QRS$ by the AA ~ Postulate. Then use the proportion $\frac{QX}{QR} = \frac{QY}{QS}$ and the given proportion $\frac{AB}{QR} = \frac{AC}{QS}$ to show that $AC = QY$. Then prove that $\triangle ABC \cong \triangle QXY$. Finally, prove that $\triangle ABC \sim \triangle QRS$ by the AA ~ Postulate.



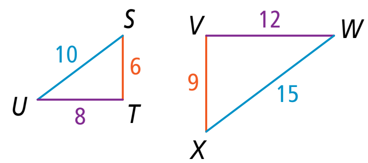
Problem 2 Verifying Triangle Similarity

Are the triangles similar? If so, write a similarity statement for the triangles.

A



Use the side lengths to identify corresponding sides. Then set up ratios for each pair of corresponding sides.



Shortest sides $\quad \frac{ST}{XV} = \frac{6}{9} = \frac{2}{3}$

Longest sides $\quad \frac{US}{WX} = \frac{10}{15} = \frac{2}{3}$

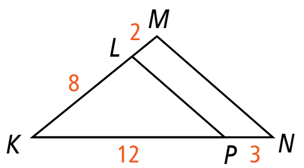
Remaining sides $\quad \frac{TU}{VW} = \frac{8}{12} = \frac{2}{3}$

All three ratios are equal, so corresponding sides are proportional.
$\triangle STU \sim \triangle XVW$ by the SSS ~ Theorem.

Plan

How can you make it easier to identify corresponding sides and angles?
Sketch and label two separate triangles.

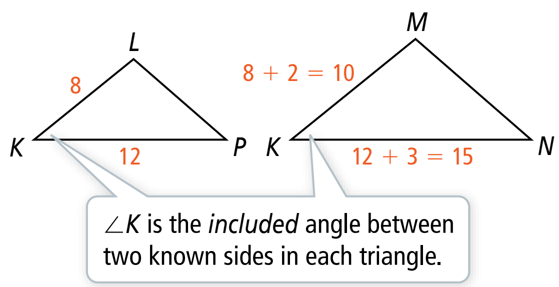
B



$\angle K \cong \angle K$ by the Reflexive Property of Congruence.

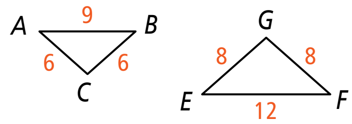
$\frac{KL}{KM} = \frac{8}{10} = \frac{4}{5}$ and $\frac{KP}{KN} = \frac{12}{15} = \frac{4}{5}$.

So, $\triangle KLP \sim \triangle KMN$ by the SAS ~ Theorem.

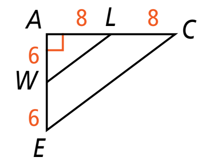


Got It? 2. Are the triangles similar? If so, write a similarity statement for the triangles and explain how you know the triangles are similar.

a.



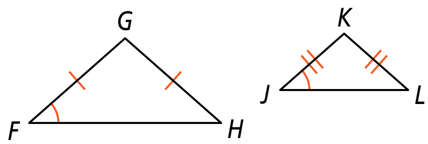
b.



Proof Problem 3 Proving Triangles Similar

Given: $\overline{FG} \cong \overline{GH}$,
$\overline{JK} \cong \overline{KL}$,
$\angle F \cong \angle J$

Prove: $\triangle FGH \sim \triangle JKL$



Know
The triangles are isosceles, so the base angles are congruent.

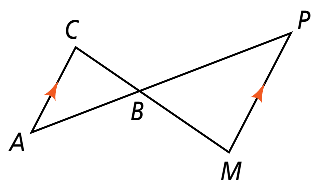
Need
You need to show that the triangles are similar.

Plan
Find two pairs of corresponding congruent angles and use the AA ~ Postulate to prove the triangles are similar.

Statements	Reasons
1) $\overline{FG} \cong \overline{GH}$, $\overline{JK} \cong \overline{KL}$	**1)** Given
2) $\triangle FGH$ is isosceles. $\triangle JKL$ is isosceles.	**2)** Def. of an isosceles $\triangle$
3) $\angle F \cong \angle H$, $\angle J \cong \angle L$	**3)** Base $\angle$s of an isosceles $\triangle$ are $\cong$.
4) $\angle F \cong \angle J$	**4)** Given
5) $\angle H \cong \angle J$	**5)** Transitive Property of $\cong$
6) $\angle H \cong \angle L$	**6)** Transitive Property of $\cong$
7) $\triangle FGH \sim \triangle JKL$	**7)** AA ~ Postulate

Got It? 3. a. Given: $\overline{MP} \parallel \overline{AC}$

Prove: $\triangle ABC \sim \triangle PBM$



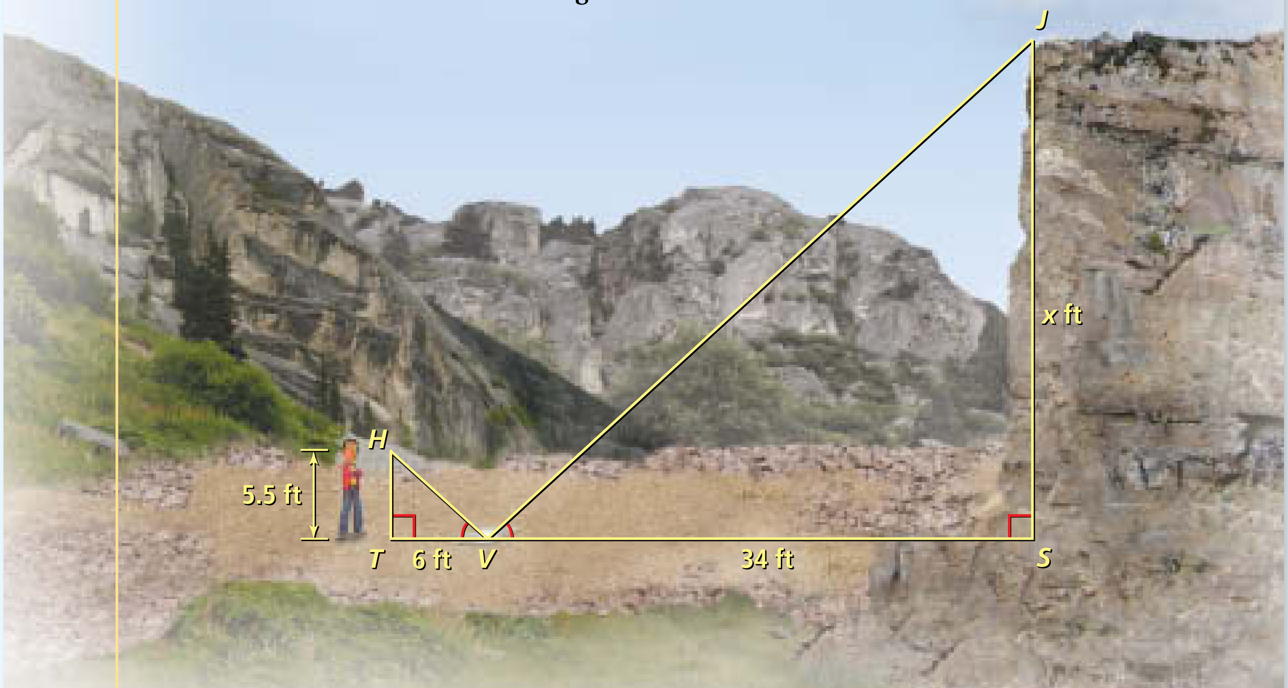
b. Reasoning For the figure at the right, suppose you are given only that $\frac{CA}{PM} = \frac{CB}{MB}$. Could you prove that the triangles are similar? Explain.

Essential Understanding Sometimes you can use similar triangles to find lengths that cannot be measured easily using a ruler or other measuring device.

You can use **indirect measurement** to find lengths that are difficult to measure directly. One method of indirect measurement uses the fact that light reflects off a mirror at the same angle at which it hits the mirror.

Problem 4 Finding Lengths in Similar Triangles

Rock Climbing **Before rock climbing, Darius wants to know how high he will climb. He places a mirror on the ground and walks backward until he can see the top of the cliff in the mirror. What is the height of the cliff?**



Plan

Before solving for x, verify that the triangles are similar. $\triangle HTV \sim \triangle JSV$ by the AA~ Postulate because $\angle T \cong \angle S$ and $\angle HVT \cong \angle JVS$.

$\triangle HTV \sim \triangle JSV$	AA ~ Postulate
$\frac{HT}{JS} = \frac{TV}{SV}$	Corresponding sides of ~ triangles are proportional.
$\frac{5.5}{x} = \frac{6}{34}$	Substitute.
$187 = 6x$	Cross Products Property
$31.2 \approx x$	Solve for x.

The cliff is about 31 ft high.

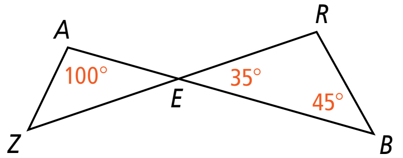
Got It? 4. Reasoning Why is it important that the ground be flat to use the method of indirect measurement illustrated in Problem 4? Explain.

Lesson Check

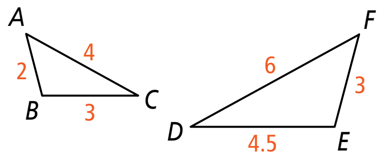
Do you know HOW?

Are the triangles similar? If yes, write a similarity statement and explain how you know they are similar.

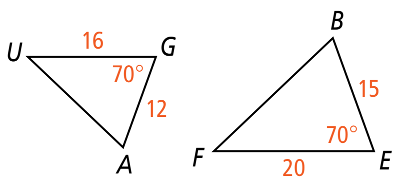
1.



2.



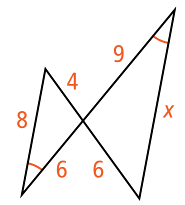
3.



Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary How could you use indirect measurement to find the height of the flagpole at your school?

5. Error Analysis Which solution for the value of x in the figure at the right is *not* correct? Explain.



A.

$$\frac{4}{8} = \frac{8}{x}$$
$$4x = 72$$
$$x = 18$$

B.

$$\frac{8}{x} = \frac{4}{6}$$
$$48 = 4x$$
$$12 = x$$

6. a. Compare and Contrast How are the SAS Similarity Theorem and the SAS Congruence Postulate alike? How are they different?

b. How are the SSS Similarity Theorem and the SSS Congruence Postulate alike? How are they different?

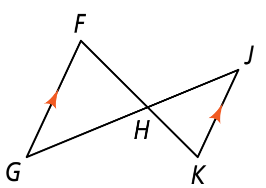
Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

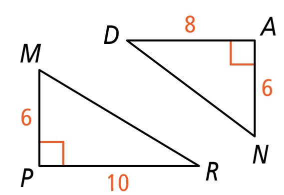
Determine whether the triangles are similar. If so, write a similarity statement and name the postulate or theorem you used. If not, explain.

See Problems 1 and 2.

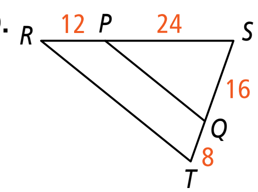
7.



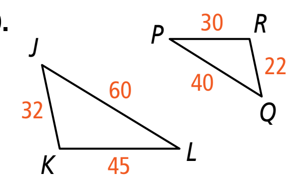
8.



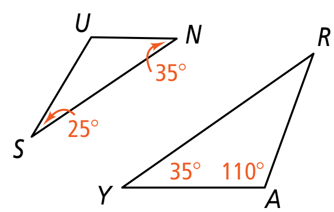
9.



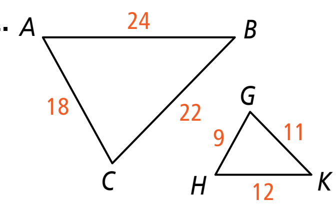
10.



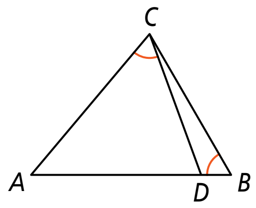
11.



12.

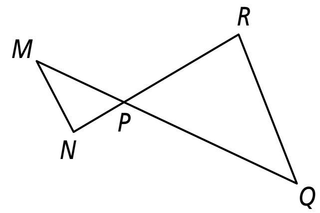


13. Given: $\angle ABC \cong \angle ACD$
Proof **Prove:** $\triangle ABC \sim \triangle ACD$



14. Given: $PR = 2NP$,
Proof $PQ = 2MP$
Prove: $\triangle MNP \sim \triangle QRP$

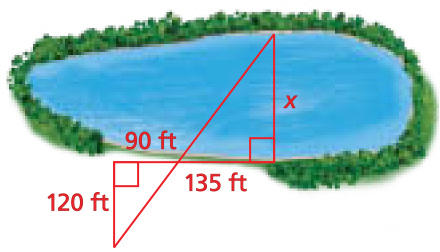
See Problem 3.



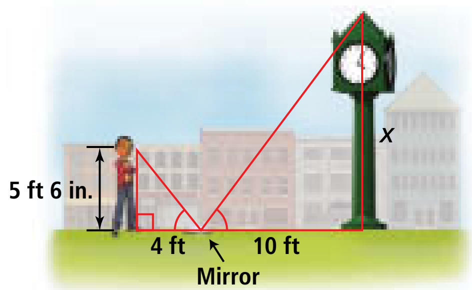
Indirect Measurement **Explain why the triangles are similar. Then find the distance represented by *x*.**

See Problem 4.

15.



16.

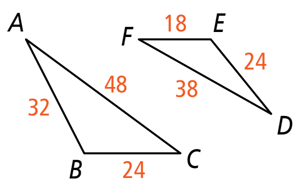


17. Washington Monument At a certain time of day, a 1.8-m-tall person standing next to the Washington Monument casts a 0.7-m shadow. At the same time, the Washington Monument casts a 65.8-m shadow. How tall is the Washington Monument?

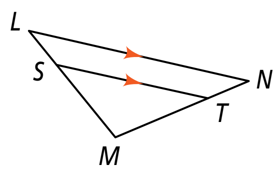
B Apply

Can you conclude that the triangles are similar? If so, state the postulate or theorem you used and write a similarity statement. If not, explain.

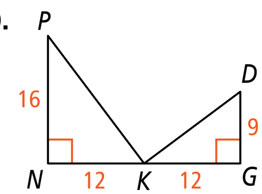
18.



19.



20.



21. a. Are two isosceles triangles always similar? Explain.
b. Are two right isosceles triangles always similar? Explain.

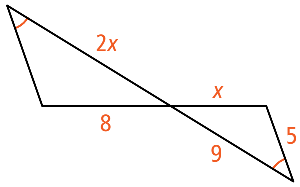
22. Think About a Plan On a sunny day, a classmate uses indirect measurement to find the height of a building. The building's shadow is 12 ft long and your classmate's shadow is 4 ft long. If your classmate is 5 ft tall, what is the height of the building?

- Can you draw and label a diagram to represent the situation?
- What proportion can you use to solve the problem?

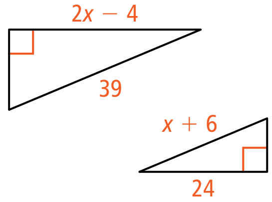
23. Indirect Measurement A 2-ft vertical post casts a 16-in. shadow at the same time a nearby cell phone tower casts a 120-ft shadow. How tall is the cell phone tower?

Algebra **For each pair of similar triangles, find the value of x.**

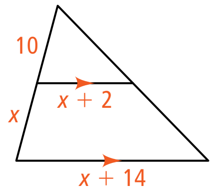
24.



25.

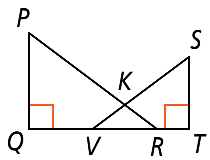


26.



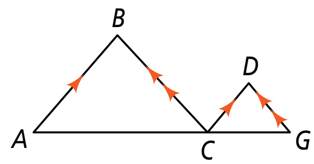
27. Proof **Given:** $\overline{PQ} \perp \overline{QT}$, $\overline{ST} \perp \overline{TQ}$, $\frac{PQ}{ST} = \frac{QR}{TV}$

Prove: $\triangle VKR$ is isosceles.



28. Proof **Given:** $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DG}$

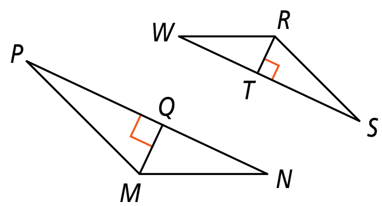
Prove: $AB \cdot CG = CD \cdot AC$



29. Reasoning Does any line that intersects two sides of a triangle and is parallel to the third side of the triangle form two similar triangles? Justify your reasoning.

30. Constructions Draw any $\triangle ABC$ with $m\angle C = 30$. Use a straightedge and compass to construct $\triangle LKJ$ so that $\triangle LKJ \sim \triangle ABC$.

31. Reasoning In the diagram at the right, $\triangle PMN \sim \triangle SRW$. $\overline{MQ}$ and $\overline{RT}$ are altitudes. The scale factor of $\triangle PMN$ to $\triangle SRW$ is 4 : 3. What is the ratio of $\overline{MQ}$ to $\overline{RT}$? Explain how you know.

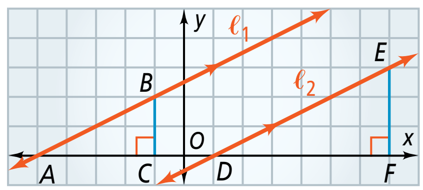


32. Coordinate Geometry Proof $\triangle ABC$ has vertices $A(0, 0)$, $B(2, 4)$, and $C(4, 2)$. $\triangle RST$ has vertices $R(0, 3)$, $S(-1, 5)$, and $T(-2, 4)$. Prove that $\triangle ABC \sim \triangle RST$. (*Hint:* Graph $\triangle ABC$ and $\triangle RST$ in the coordinate plane.)

33. Proof Write a proof of the following: Any two nonvertical parallel lines have equal slopes.

Given: Nonvertical lines ℓ_1 and ℓ_2, $\ell_1 \parallel \ell_2$, $\overline{EF}$ and $\overline{BC}$ are $\perp$ to the x-axis

Prove: $\frac{BC}{AC} = \frac{EF}{DF}$



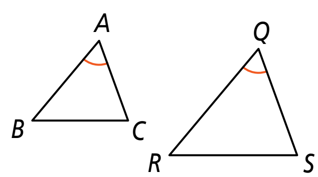
34. Proof Use the diagram in Exercise 33. Prove: Any two nonvertical lines with equal slopes are parallel.

Challenge

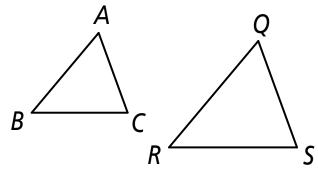
35. Proof Prove the Side-Angle-Side Similarity Theorem (Theorem 7-1).

Given: $\frac{AB}{QR} = \frac{AC}{QS}$, $\angle A \cong \angle Q$

Prove: $\triangle ABC \sim \triangle QRS$



36. Prove the Side-Side-Side Similarity Theorem (Theorem 7-2).
Proof
Given: $\frac{AB}{QR} = \frac{AC}{QS} = \frac{BC}{RS}$
Prove: $\triangle ABC \sim \triangle QRS$

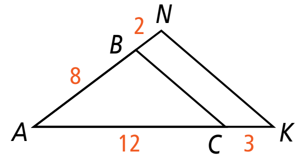


Standardized Test Prep

SAT/ACT

37. Complete the statement $\triangle ABC \sim$ __?__. By which postulate or theorem are the triangles similar?

Ⓐ $\triangle AKN$; SSS ~
Ⓑ $\triangle AKN$; SAS ~
Ⓒ $\triangle ANK$; SAS ~
Ⓓ $\triangle ANK$; AA ~



38. $\angle 1$ and $\angle 2$ are alternate interior angles formed by two parallel lines and a transversal. If $m\angle 2 = 68$, what is $m\angle 1$?

Ⓕ 22 Ⓖ 68 Ⓗ 112 Ⓘ 122

39. The length of a rectangle is twice its width. If the perimeter of the rectangle is 72 in., what is the length of the rectangle?

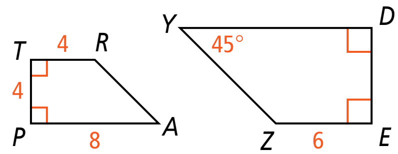
Ⓐ 12 in. Ⓑ 18 in. Ⓒ 24 in. Ⓓ 36 in.

Extended Response

40. Graph $A(2, 4)$, $B(4, 6)$, $C(6, 4)$, and $D(4, 2)$. What type of polygon is $ABCD$? Justify your answer.

Mixed Review

***TRAP* ~ *EZYD*. Use the diagram at the right to find the following.**



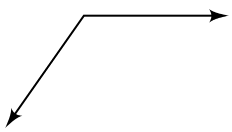
See Lesson 7-2.

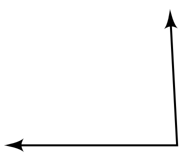
41. the scale factor of *TRAP* to *EZYD*

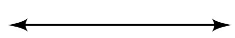
42. $m\angle R$ **43.** DY **44.** $\frac{DE}{PT}$

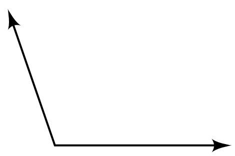
Use a protractor to find the measure of each angle. Classify the angle as *acute*, *right*, *obtuse*, or *straight*.

See Lesson 1-4.

45.


46.


47.


48.


Get Ready! To prepare for Lesson 7-4, do Exercises 49–52.

Algebra Identify the means and extremes of each proportion. Then solve for x.

See Lesson 7-1.

49. $\frac{x}{8} = \frac{18}{24}$ **50.** $\frac{12}{m} = \frac{18}{20}$ **51.** $\frac{15}{x + 2} = \frac{9}{x}$ **52.** $\frac{x - 3}{x + 4} = \frac{5}{9}$